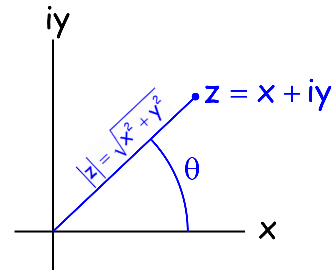
Essential Modern Physics Knowledge

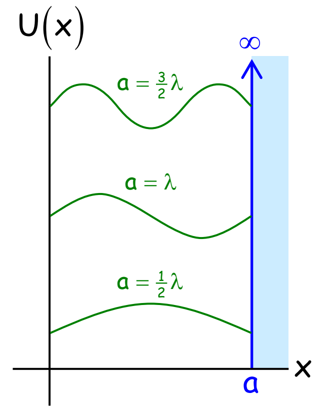


Complex Numbers

For $z = x + iy$, the complex conjugate is $z^* = x - iy$ and the absolute value is the distance on the complex plane from the origin to the point z.

$$|z|^2 = zz^* = (x+iy)(x-iy) = x^2 + y^2$$

We also utilize Euler's formula stating that $e^{\pm i\theta} = \cos\theta \pm i\sin\theta$.



Particle in a box: allowed energy states

For a quantum particle of energy, E, in a potential box where U(x) = 0 for 0 < x < a, it can only exist between x = 0 and x = a (since E = K + U and U = ∞ outside of 0 < x < a). **A quantum particle is described by a wave function** that can exist in the box at only certain wavelengths (λ, first three shown) the wave function must be zero at the sides (x = 0 & x = a).

For a standing wave, $\psi(x) = A\sin(kx) + B\cos(kx)$, this requires that

$ka = n\pi$ where n = 1, 2, 3, ... and $k = \frac{2\pi}{\lambda}$ is the wave number

**** Other than saying a quantum particle is described by a wave, this is just math! ****

The physics comes in with de Broglie:

$$p = \frac{h}{\lambda} = \hbar k \quad \text{for} \quad k = \frac{2\pi}{\lambda} \quad \text{and} \quad \hbar = \frac{h}{2\pi}$$

KNOW THESE RELATIONSHIPS!

Thus, the allowed momenta of the particle in the box are:

$$a = \frac{n}{2}\lambda = \frac{n\pi}{k} = \frac{n\pi\hbar}{p} \Rightarrow p = \frac{n\pi\hbar}{a}$$

BE ABLE TO FIGURE THESE OUT

Since the energy of the particle is purely kinetic (U = 0), $E = p^2/2m$ gives the allowed energies:

BE ABLE TO DERIVE THIS.

$$E = n^2 \frac{\pi^2\hbar^2}{2ma^2}, \quad n = 1, 2, 3, \ldots$$

TZDII (7.23)

Probability Density, Normalization, & Expectation Value

$$|\Psi(\vec{r},t)|^2 = \text{probability (volume) density for finding particle at } \vec{r}$$

TZDII (6.15)

Prob. of finding particle between x_1 & x_2 $= \int_{x_1}^{x_2} |\psi(x)|^2 dx \approx |\psi(x = x_1)|^2 \Delta x$

UNDERSTAND WHY INTEGRAL IS APPROXIMATED BY A PRODUCT.

Normalization

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

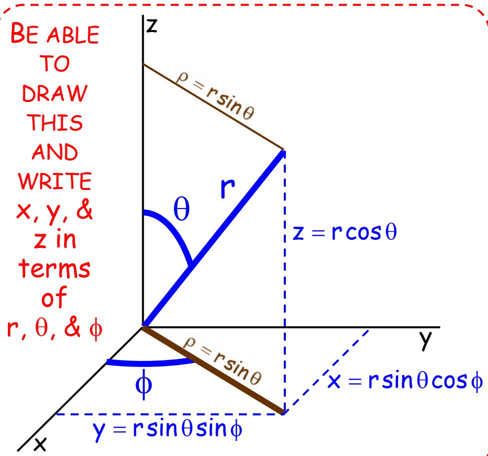
TZDII (7.55)

CARTESIAN COORDINATES GO FROM -∞ TO +∞

The expectation value (value expected after many measurements) of f(x) with a probability density $\left|\psi(x)\right|^2$ is

$$\int f(x)\left|\psi(x)\right|^2 dx = \int f(x)p(x)dx \qquad \text{TZDII (7.69)}$$

BE ABLE TO DRAW THIS AND WRITE x, y, & z in terms of r, θ, & ϕ



3-D SCHRÖDINGER EQUATION

For the hydrogen, we assume a purely radial potential due to the charge of the proton (PE = force x distance).

$$U(r) = -\frac{ke^2}{r}$$

The electron's energy can only be multiples of the Rydberg Energy as shown in TZDII equations 5.22 and 5.23.

$$E = -\frac{m_e\left(ke^2\right)^2}{2\hbar^2}\frac{1}{n^2} = -\frac{E_R}{n^2} = -\frac{13.6}{n^2}\text{eV}$$

KNOW THIS.

Separation of Variables

Assume that the wave function of the electron can be written as a product

$$\psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$$

Substituting this into the Schrödinger equation and setting

$$\text{Function of } \phi = \text{Functions of r and } \theta = -m^2$$

$$\text{Function of } \theta = \text{Function of r} = -k = -\ell(\ell+1)$$

HAVE A CONCEPTUAL UNDERSTANDING OF THIS PROCESS.

Yields three differential equations, one in each variable that can be solved for various values of n, m and ℓ.

The ϕ and θ solutions depending on m and ℓ are the Spherical Harmonics with the θ solutions given in Table 8.1 as the Associated Legendre Functions.

The R solutions depending on n and ℓ are given in table 8.2. The normalization of these equations requires that the electron be found within a spherical volume, thus the differential volume element (supplied by the normalization of the θ and ϕ solutions in the full wave function) becomes $4\pi r^2 dr$, giving

$$\text{Prob. of finding } e^- \text{ in a spherical volume } = \int_{r_1}^{r_2} 4\pi r^2\left|R(r)\right|^2 dr$$

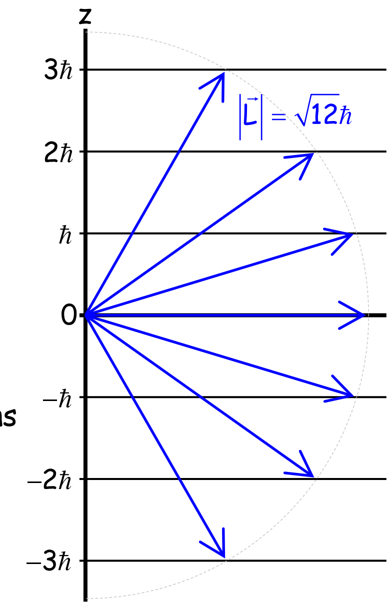
Normalization

$$\int_0^\infty 4\pi r^2\left|R(r)\right|^2 dr = 1 \qquad \text{TZDII (8.84)}$$

r IN SPHERICAL GOES FROM 0 TO ∞

Know how to demonstrate a function is a solution to a differential equation.

QUANTUM NUMBERS GIVE PHYSICAL QUANTITIES (THAT WE CAN MEASURE IN THE LAB!)



Principle Quantum Number, n gives the energy of a state:

n = 1, 2, 3, …

$$E_n = -\frac{1}{2}\frac{m_e\left(ke^2\right)^2}{n^2\hbar^2} = \frac{-13.6\text{ eV}}{n^2}$$

Angular Momentum Quantum Number, ℓ gives magnitude of the angular momentum:

$$\left|\vec{L}\right| = \hbar\sqrt{\ell\left(\ell+1\right)}$$

ℓ = 0, 1, 2, 3, …, (n − 1)

The orbitals are named for the ℓ values,

- s: ℓ = 0 = sharp
- p: ℓ = 1 = principle
- d: ℓ = 2 = diffuse
- f: ℓ = 3 = fundamental

} Named for spectral line classifications

Magnetic Quantum Number, m gives the z-component of the angular momentum:

m = -ℓ, …, 0, … ,ℓ

$$L_z = m\hbar$$

Spin Quantum Number, m_s gives the z-component of the spin angular momentum:

$m_s = \pm\frac{1}{2}$ $\quad S_z = m_s\hbar$

Magnitude of the Spin Angular Momentum is given by S and s:

$$\left|\vec{S}\right| = \hbar\sqrt{s\left(s+1\right)} = \frac{\sqrt{3}}{2}\hbar, \quad s = \frac{1}{2}$$

ZEEMAN EFFECTS

Zeeman Effect: The electron's orbital momentum responds to an external magnetic field
- requires ℓ ≥ 1 AND total electron spin = 0 (singlet states)

Anomalous Zeeman Effect: The electron's spin responds to an external magnetic field
- occurs for all values of ℓ

Fine Structure: The e^-'s spin responds to magnetic field of p^+'s "orbit"
- "spin orbit coupling"

Hyperfine Structure: The e^-'s spin responds to magnetic field of p^+'s spin
- "spin flip" transition gives 1420 MHz HI line (21 cm line)